Angel • Runde

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Intermediate

Algebra

for College Students

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Intermediate Algebra

for College Students

10e

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P Pearson

To my wife, Kathy,
and our sons, Robert and Steven
Allen R. Angel

To my wife, Kristin,
and our sons, Alex, Nick, and Max
Dennis C. Runde

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Brief Contents

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Preface

Welcome to the 10th edition of *Intermediate Algebra for College Students!* This book has been used by thousands of students and other adults who have never been exposed to algebra or those who have been exposed but need a refresher course. Our primary goal was to write a book that students can read, understand, and enjoy. To achieve this goal we have used short sentences, clear explanations, and many detailed, worked-out examples. We have tried to make the book relevant to college students by using practical applications of algebra throughout the text.

New to This Edition

One of the most important features of the text is its emphasis on readability. The book is very understandable to students at all reading skill levels. The Tenth Edition retains this emphasis and has been revised with a focus on improving accessibility and addressing the learning needs and styles of today's students. To this end, the following changes have been made:

Content Changes

- We've done an extensive review of exercise sets, including an analysis of data analytics on exercise usage, leading to modification of exercises and exercise sets as follows:
  - Exercise sets have been modified to ensure precise graduation from simple to more complex and include more direct matching of the book examples and the corresponding exercises in MyLab Math. This creates a better experience throughout for students as well as making the material in the book better connected to the homework students do.
  - Precise correlation has been made between each odd and even exercise. The odds can be used as examples and solutions are provided, and the evens can be assigned as homework or in MyLab Math.
  - Now Try Exercises are revised, with particular focus on odd–even pairing.
- Chapter openers each include a new video, created by the authors, that explains how the material presented in the chapter is used to solve problems from everyday life. These explanations are carried into the actual solution to one or more exercises that are in the chapter and to other assignable exercises.
- Renewed focus on the Understanding Algebra feature throughout the book. Many Understanding Algebra boxes are new or revised for greater clarity. The new design of the Understand Algebra boxes will make them stand out more.

- The MyLab Math course itself includes extensive enhancements to improve outcomes for students:
  - The addition of Skill Builder exercises
  - Author-developed Sample Assignments that instructors can assign to utilize all of the new exercise enhancements
  - Learning Catalytics can be accessed from the MyLab Math course
  - Fully accessible PowerPoint slides

Features of the Text

Accuracy
Accuracy in a mathematics text is essential. To ensure accuracy in this book, math teachers from around the country have read the pages carefully for typographical errors and have checked all the answers.

Making Connections
Many of our students do not thoroughly grasp new concepts the first time they are presented. In this text we encourage students to make connections. That is, we introduce a concept, then later in the text briefly reintroduce it and build upon it. Often an important concept is used in many sections of the text. Important concepts are also reinforced throughout the text in the Cumulative Review Exercises and Cumulative Review Tests.

Chapter Opening Application
Each chapter begins with a real-life application related to the material covered in the chapter and further illuminated through an author-created video explanation within MyLab Math. By the time students complete the chapter, they should have the knowledge to work the problem.

Goal of This Chapter
This feature on the chapter opener page gives students a preview of the chapter and also indicates where this material will be used again in other chapters of the book. This material helps students see the connections among various topics in the book and the connection to real-world situations.

Keyed Section Objectives
Each section opens with a list of skills that the student should learn in that section. The objectives are then keyed to the appropriate portions of the sections with blue numbers such as **1**.

Problem Solving

Pólya's five-step problem-solving procedure is discussed in Section 1.2. Throughout the book, problem solving and Pólya's problem-solving procedure are emphasized.

Practical Applications

Practical applications of algebra are stressed throughout the text. Students need to learn how to translate application problems into algebraic symbols. The problem-solving approach used throughout this text gives students ample practice in setting up and solving application problems. The use of practical applications motivates students.

Detailed, Worked-Out Examples

A wealth of examples have been worked out in a step-by-step, detailed manner. Important steps are highlighted in color, and no steps are omitted until after the student has seen a sufficient number of similar examples.

Now Try Exercises

In each section, after each example, students are asked to work an exercise that parallels the example given in the text. These Now Try Exercises make the students *active,* rather than passive, learners and they reinforce the concepts as students work the exercises. Through these exercises, students have the opportunity to immediately apply what they have learned. After each example, Now Try Exercises are indicated in orange type such as Now Try Exercise 27 . They are also indicated in green type in the exercise sets, such as 27.

Study Skills Section

Students taking this course may benefit from a review of essential study skills. Such study skills are essential for success in mathematics. Section 1.1, the first section of the text, discusses such study skills. This section should be very beneficial for your students and should help them to achieve success in mathematics.

Understanding Algebra

Understanding Algebra boxes appear in the margin throughout the text. Placed at key points, Understanding Algebra boxes help students focus on the important concepts and facts that they need to master.

Helpful Hints

The Helpful Hint boxes offer useful suggestions for problem solving and other varied topics. They are set off in a special manner so that students will be sure to read them.

Avoiding Common Errors

Common student errors are illustrated. Explanations of why the shown procedures are incorrect are given. Explanations of how students may avoid such errors are also presented.

Exercise Sets

The exercise sets are broken into three main categories: Warm-Up Exercises, Practice the Skills, and Problem Solving. Many exercise sets also contain Concept/Writing Exercises, Challenge Problems, and/or Group Activities. Each exercise set is graded in difficulty, and the exercises are paired. The early problems help develop the students' confidence, and then students are eased gradually into the more difficult problems. A sufficient number and variety of examples are given in each section for students to successfully complete even the more difficult exercises. The number of exercises in each section is more than ample for student assignments and practice.

Warm-Up Exercises

The exercise sets begin with Warm-Up Exercises. These fill-in-the-blank exercises include an emphasis on vocabulary. They serve as a great warm-up to the homework exercises or as 5-minute quizzes.

Practice the Skills Exercises

The Practice the Skills exercises reinforce the concepts and procedures discussed in the section. These exercises provide students with practice in working problems similar to the examples given in the text. In many sections the Practice the Skills exercises are the main and most important part of the exercise sets.

Problem-Solving Exercises

These exercises help students become better thinkers and problem solvers. Many of these exercises involve real-life applications of algebra. It is important for students to be able to apply what they learn to real-life situations. Many problem-solving exercises help with this.

Concept/Writing Exercises

Most exercise sets include exercises that require students to write out the answers in words. These exercises improve students' understanding and comprehension of the material. Many of these exercises involve problem solving and conceptualization and help develop better reasoning and critical thinking skills.

Challenge Problems

These exercises, which are part of many exercise sets, provide a variety of problems. Many were written to stimulate student thinking. Others provide additional applications of algebra or present material from future sections of the book so that students can see and learn the material on their own before it is covered in class. Others are more challenging than those in the regular exercise set.

Group Activities

Many exercise sets have Group Activity exercises that lead to interesting group discussions. Many students

learn well in a cooperative learning atmosphere, and these exercises will get students talking mathematics to one another.

Cumulative Review Exercises

All exercise sets (beginning with Section 1.3) contain questions from previous sections in the chapter and from previous chapters. These Cumulative Review Exercises will reinforce topics that were previously covered and help students retain the earlier material while they are learning the new material. For the students' benefit, Cumulative Review Exercises are keyed to the section where the material is covered, using brackets, such as [3.4].

Mid-Chapter Tests

In the middle of each chapter is a Mid-Chapter Test. Students should take each Mid-Chapter Test to make sure they understand the material presented in the chapter up to that point. In the student answers, brackets such as [2.3] are used to indicate the section where the material was first presented.

Chapter Summary

At the end of each chapter is a comprehensive chapter summary that includes important chapter facts and examples illustrating these important facts.

Chapter Review Exercises

At the end of each chapter are review exercises that cover all types of exercises presented in the chapter. The review exercises are keyed using colored numbers and brackets, such as [1.5], to the sections where the material was first introduced.

Chapter Practice Tests

The comprehensive end-of-chapter practice tests enable students to see how well they are prepared for the actual class test. The section where the material was first introduced is indicated in brackets in the student answers.

Cumulative Review Tests

These tests, which appear at the end of each chapter after the first, test the students' knowledge of material from the beginning of the book to the end of that chapter. Students can use these tests for review, as well as for preparation for the final exam. These exams, like the Cumulative Review Exercises, serve to reinforce topics taught earlier. In the answer section, after each answer, the section where that material was covered is given using brackets.

Answers

The *odd-numbered answers* are provided for the exercise sets. *All answers* are provided for the Cumulative Review Exercises, Mid-Chapter Test, Chapter Review Exercises, Chapter Practice Tests, and Cumulative Review Tests. Answers are not provided to students for the Group Activity exercises because we want students to reach agreement by themselves on the answers to these exercises.

Prerequisite

The prerequisite for this course is a working knowledge of elementary algebra. Although some elementary algebra topics are briefly reviewed, students should have a basic understanding of elementary algebra before taking this course.

Modes of Instruction

The format and readability of this book, and its many resources and supplements, lend it to many different modes of instruction. The constant reinforcement of concepts will result in greater understanding and retention of the material by your students.

The features of the text and its supplements make it suitable for many types of instructional modes, including:

- face-to-face courses
- hybrid or blended courses
- emporium-based courses
- online instruction
- accelerated courses
- self-paced instruction
- inverted classrooms
- cooperative or group study

Resources for Success

Get the Most out of MyLab Math for
Intermediate Algebra, Tenth Edition
by Allen Angel and Dennis Runde

The Angel/Runde team has helped thousands of students learn algebra through clear examples and concise language. With this revision, the authors have continued their hallmark clear writing style. This, along with new media resources and revamped exercise sets, provides students with a comprehensive learning and practice environment in MyLab Math. Bringing the authors' voice and approach into the MyLab course gives students the motivation, understanding, and skill set they need to master algebra.

Take advantage of the following resources to get the most out of your MyLab Math course.

Support and Motivate with Video Resources

NEW! Chapter Opener Videos highlight how the math students are about to learn can be applied and used in the real world. Providing an interesting and useful overview of the chapter, these videos can be assigned or even used in the classroom to kick off a lecture.
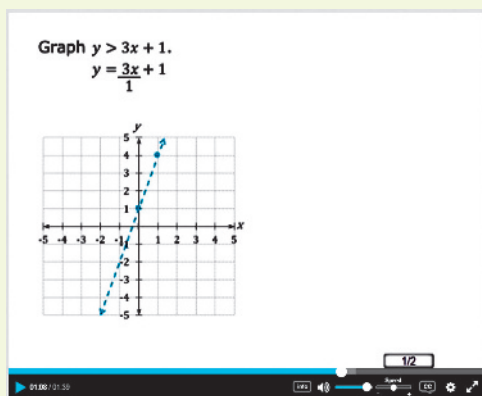
Instructional Videos walk students through concepts and examples in a modern presentation format. Videos are accessible in many ways, including from the eText pages and from within homework exercises and can also be assigned in a media assignment to encourage students to watch them. All videos can be played from any laptop or mobile device to provide support even on the go.

Chapter Test Prep Videos help students during their most teachable moment—when they are preparing for a test. The videos provide step-by-step solutions for every exercise found in the text's Chapter Tests.

pearson.com/mylab/math

Personalize Students' Learning

One size does not fit all, especially when it comes to developmental math students. Instructors have the option to personalize students' experiences in the MyLab course with new tools, including personalized homework and Skill Builder.

Personalized Homework delivers assignments to students tailored to their understanding of topics based on their performance on a test or quiz. This way, students can focus on just the topics they have not yet mastered and receive credit for the topics they mastered on the quiz or test.

New! Skill Builder assignments offer just-in-time adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. This new feature allows instructors to assign fewer questions for homework, allowing students to complete as many or as few questions needed.

Build Your Course More Easily

Enhanced Sample Assignments make course setup easier by giving instructors a starting point for each chapter. Each assignment has been carefully curated for this specific text by author Dennis Runde based on his and his students' experiences with MyLab Math and has been crafted to include a thoughtful mix of question types.

pearson.com/mylab/math

Student and Instructor Resources

STUDENT RESOURCES

Student Solutions Manual
Provides complete worked-out solutions to
- the odd-numbered section exercises
- all exercises in the Mid-Chapter Tests, Chapter Reviews, Chapter Practice Tests, and Cumulative Review Tests

ISBN: 978-0-13-479484-6

Student Workbook
- Extra practice exercises for every section of the text with ample space for students to show their work

ISBN: 978-0-13-479490-7

Video Program
The Angel/Runde video program, available through MyLab Math, includes:
- Objective-based videos
- Example-based videos covering most examples and related end of section exericses
- Chapter Test Prep videos that offer step-by-step solutions to exercises in Chapter Tests
- Videos are captioned, and can be viewed on any mobile device

INSTRUCTOR RESOURCES

Annotated Instructor's Edition
Contains all the content found in the student edition, plus the following:
- Answers to exercises on the same text page with graphing answers in the Graphing Answer section at the back of the text
- Instructor Example provided in the margin paired with each student example

Instructor's Resource Manual with Tests and Mini-Lectures
- Mini-lectures for each text section
- Several forms of test per chapter (free response and multiple choice)
- Answers to all items
- Available for download from pearson.com and in MyLab Math

Instructor's Solutions Manual
- Provides complete worked-out solutions to all section exercises
- Available for download from pearson.com and in MyLab Math

TestGen®
- Enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.
- Algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button; instructors can also modify test bank questions or add new questions.

MyLab Math Online Course (access code required)
MyLab Math is the world's leading homework, tutorial, and assessment program. It creates personalized learning experiences for students and provides powerful tools for instructors. Learn more about MyLab Math at pearson.com/mylab/math.

Acknowledgments

We thank our spouses, Kathy Angel and Kris Runde, for their support and encouragement throughout the project. We are grateful for their wonderful support and understanding while we worked on the book.

We also thank our children: Robert and Steven Angel and Alex, Nick, and Max Runde. They also gave us support and encouragement and were very understanding when we could not spend as much time with them as we wished because of book deadlines. Special thanks to daughter-in-law, Kathy; mother-in-law, Patricia; and father-in-law, Scott. Without the support and understanding of our families, this book would not be a reality.

We want to thank Brianna Kurtz, Rhea Meyerholtz, Laurie Semarne, and Hal Whipple for accuracy reviewing the pages and checking all answers.

Many people at Pearson deserve thanks, including all those listed on the copyright page. In particular, we thank Michael Hirsch, Editor-in-Chief; Rachel Reeve, Content Producer; Shannon Bushee, Editorial Assistant; Shana Siegmund, Producer; Eric Gregg, Senior Content Developer; Alicia Frankel, Product Marketing Manager; and Brooke Imbornone, Product Marketing Assistant.

We would like to thank the following reviewers and focus group participants of this Tenth Edition (marked with an asterisk) and recent editions for their thoughtful comments and suggestions.

Darla Aguilar, *Pima Community College, AZ*
Frances Alvarado, *University of Texas–Pan American, TX*
Jose Alvarado, *University of Texas–Pan American, TX*
Ben Anderson, *Darton College, GA*
Linda Barton, *Ball State, IN*
Elizabeth Bonawitz, *University of Rio Grande, OH*
Sharon Berrian, *Northwest Shoals Community College, AL*
Dianne Bolen, *Northeast Mississippi Community College, MS*
Julie Bonds, *Sonoma State University, CA*
Clark Brown, *Mojave Community College, AZ*
Connie Buller, *Metropolitan Community College, NE*
Marc D. Campbell, *Daytona State College, FL*
Julie Chesser, *Owens Community College, OH*
Kim Christensen, *Maple Woods Community College, MO*
Barry Cogan, *Macomb Community College, MI*
Pat C. Cook, *Weatherford College, TX*
Olga Cynthia Harrison, *Baton Rouge Community College, LA*
Lisa DeLong Cuneo, *Pennsylvania State University–Dobois, PA*
Stephan Delong, *Tidewater Community College, VA*
*Deborah Doucette, *Erie Community College (North), NY*
William Echols, *Houston Community College, TX*
Karen Egedy, *Baton Rouge Community College, LA*

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Daniel Fahringer, *Harrisburg Area Community College, PA*
Dale Felkins, *Arkansas Technical University, AR*
*Ernie Forgione, *Central Maine Community College*
Reginald Fulwood, *Palm Beach State College, FL*
Larry Gilligan, *University of Cincinnati, OH, Emeritus*
Susan Grody, *Broward College, FL*
Abdollah Hajikandi, *State University of New York–Buffalo, NY*
Sharon Hamsa, *Longview Community College, MO*
Cynthia Harrison, *Baton Rouge Community College, LA*
Richard Hobbs, *Mission College, CA*
Joe Howe, *St. Charles Community College, MO*
Laura L. Hoye, *Trident Technical College, SC*
Barbara Hughes, *San Jacinto Community College (Central), TX*
Mary Johnson, *Inver Hills Community College, MN*
Maryann Justinger, *Erie Community College (South), NY*
Judy Kasabian, *El Camino College, CA*
John Kawai, *Los Angeles Valley College, CA*
Jane Keller, *Metropolitan Community College, NE*
Mike Kirby, *Tidewater Community College, VA*
William Krant, *Palo Alto College, TX*
Gayle L. Krzemine, *Pikes Peak Community College, CO*
Mitchel Levy, *Broward College, FL*
Mitzi Logan, *Pitt Community College, NC*
Mary Lou Baker, *Columbia State Community College, TN*
Jason Mahar, *Monroe Community College, NY*
Kimberley A. Martello, *Monroe Community College, NY*
Constance Meade, *College of Southern Idaho, ID*
Claire Medve, *State University of New York–Canton, NY*
Lynnette Meslinsky, *Erie Community College, NY*
Elizabeth Morrison, *Valencia College, FL*
Elsie Newman, *Owens Community College, OH*
Charlotte Newsom, *Tidewater Community College, VA*
Charles Odion, *Houston Community College, TX*
Jean Olsen, *Pikes Peak Community College, CO*
Jearme Pirie, *Erie Community College (North), NY*
Behnaz Rouhani, *Athens Technical College, GA*
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Cristela Sifuentez, *University of Texas–Pan American, TX*
Fereja Tahir, *Illinois Central College, IL*
Burnette Thompson, Jr., *Houston Community College, TX*
Mary Vachon, *San Joaguin Delta College, CA*
Andrea Vorwark, *Maple Woods Community College, MO*
Christopher Yarish, *Harrisburg Area Community College, PA*
Ronald Yates, *Community College of Southern Nevada, NY*

To the Student

Algebra is a course that requires active participation. You must read the text and pay attention in class, and, most importantly, you must work the exercises. The more exercises you work, the better.

The text was written with you in mind. Short, clear sentences are used, and many examples are given to illustrate specific points. The text stresses useful applications of algebra. Hopefully, as you progress through the course, you will come to realize that algebra is not just another math course that you are required to take, but a course that offers a wealth of useful information and applications.

The boxes marked **Understanding Algebra** should be studied carefully. They emphasize concepts and facts that you need to master to succeed. **Helpful Hints** should be studied carefully, for they stress important information. Be sure to study **Avoiding Common Errors** boxes. These boxes point out common errors and provide the correct procedures for doing these problems.

After each example you will see a Now Try Exercise reference, such as **Now Try Exercise 27**. The exercise indicated is very similar to the example given in the book. You may wish to try the indicated exercise after you read the example to make sure you truly understand the example. In the exercise set, the Now Try exercises are written in green, such as 27.

Each objective is accompanied by a video lecture that covers the concepts discussed in that section, as well as additional example problems. These videos may be accessed through MyLab Math.

Some questions you should ask your professor early in the course include: What supplements are available for use? Where can help be obtained when the professor is not available? Supplements that may be available include the Student Solutions Manual; the objective videos; and the Chapter Test Prep Videos, all of which are available from within this book's MyLab Math course. All these items are discussed under the heading of Supplements in Section 1.1 and listed in the Preface.

You may wish to form a study group with other students in your class. Many students find that working in small groups provides an excellent way to learn the material. By discussing and explaining the concepts and exercises to one another, you reinforce your own understanding. Once guidelines and procedures are determined by your group, make sure to follow them.

One of the first things you should do is to read Section 1.1, Study Skills for Success in Mathematics. Read this section slowly and carefully, and pay particular attention to the advice and information given. Occasionally, refer back to this section. This could be the most important section of the book. Pay special attention to the material on doing your homework and on attending class.

At the end of all exercise sets (beginning with Section 1.3) are **Cumulative Review Exercises.** You should work these problems on a regular basis, even if they are not assigned. These problems are from earlier sections and chapters of the text, and they will refresh your memory and reinforce those topics. If you have a problem when working these exercises, read the appropriate section of the text or study your notes that correspond to that material. The section of the text where the Cumulative Review Exercise was introduced is indicated in brackets, [ ], to the left of the exercise. After reviewing the material, if you still have a problem, make an appointment to see your professor. Working the Cumulative Review Exercises throughout the semester will also help prepare you to take your final exam.

Near the middle of each chapter is a **Mid-Chapter Test.** You should take each Mid-Chapter Test to make sure you understand the material up to that point. The section where the material was first introduced is given in brackets after the answer in the answer section of the book.

At the end of each chapter are a **Chapter Summary, Chapter Review Exercises,** a **Chapter Practice Test,** and a **Cumulative Review Test.** Before each examination you should review this material carefully and take the Chapter Practice Test (you may want to review the *Chapter Test Prep Videos* also). If you do well on the Chapter Practice Test, you should do well on the class test. The questions in the Review Exercises are marked to indicate the section in which that material was first introduced. If you have a problem with a Review Exercise question, reread the section indicated. You may also wish to take the Cumulative Review Test that appears at the end of every chapter (starting with Chapter 2).

In the back of the text there is an **answer section** that contains the answers to the *odd-numbered* exercises, including the Challenge Problems. Answers to *all* Cumulative Review Exercises, Mid-Chapter Tests, Chapter Review Exercises, Chapter Practice Tests, and Cumulative Review Tests are provided. Answers to the Group Activity exercises are not provided, for we wish students to reach agreement by themselves on answers to these exercises. The answers should be used only to check your work. For the Mid-Chapter Tests, Chapter Practice Tests, and Cumulative Review Tests, after each answer the section number where that type of exercise was covered is provided.

We have tried to make this text as clear and error free as possible. No text is perfect, however. If you find an error in the text, or an example or section that you believe can be improved, we would greatly appreciate hearing from you. If you enjoy the text, we would also appreciate hearing from you. You can submit comments to math@pearson.com, subject for Allen Angel and Dennis Runde.

Allen R. Angel
Dennis C. Runde

1 Basic Concepts

Have you ever asked yourself, "When am I going to use algebra?" In this chapter and throughout the book, we use algebra to study many real-life applications. The applications include determining the stopping distance of a car in Example 10 on page 35, calculating the annual profit of a boat-detailing business in Exercise 124 on page 39, and determining the time it would take a spacecraft to reach another star in Exercise 82 on page 56. Throughout this textbook, we will discuss many interesting ways that mathematics can be used every day.

Goals of This Chapter

In this chapter, we review algebra concepts that are central to your success in this course. Throughout this chapter, and in the entire book, we use real-life examples to show how mathematics is relevant in your daily life. In Section 1.1, we present some advice to help you establish effective study skills and habits. Other topics discussed in this chapter are sets, real numbers, and exponents.

1.1 Study Skills for Success in Mathematics and Using a Calculator

1 Have a positive attitude.

2 Prepare for and attend class.

3 Prepare for and take examinations.

4 Find help.

5 Learn to use a calculator.

You need to acquire certain study skills that will help you to complete this course successfully. These study skills will also help you succeed in any other mathematics courses you may take.

It is important for you to realize that this course is the foundation for more advanced mathematics courses. If you have a thorough understanding of algebra, you will find it easier to be successful in later mathematics courses.

1 Have a Positive Attitude

Many students may feel uneasy about learning mathematics or in some cases may experience what is often called *math anxiety*. Such feelings are quite common among students. If you find yourself feeling this way, know that you are not alone. One of our goals for this textbook is to make mathematics more understandable and less intimidating. To help students learn mathematics, we encourage students to keep an open mind and try to develop a positive attitude toward learning mathematics.

Based on past experiences in mathematics, you may feel this will be difficult. However, mathematics is something you need to work at. Many of you taking this course are more mature now than when you took previous mathematics courses. Your maturity and your desire to learn are extremely important and can make a tremendous difference in your ability to succeed in mathematics. We believe you can be successful in this course, but you also need to believe it.

2 Prepare for and Attend Class

Preview the Material   Before class, you should spend a few minutes previewing any new material in the textbook. You do not have to understand everything you read yet. Just get a feeling for the definitions and concepts that will be discussed. This quick preview will help you to understand what your instructor is explaining during class. After the material is explained in class, read the corresponding sections of the text slowly and carefully, word by word.

Read the Text   A mathematics text is not a novel. Mathematics textbooks should be read slowly and carefully. If you do not understand what you are reading, reread the material. When you come across a new concept or definition, you may wish to underline or highlight it so that it stands out. This way, when you look for it later, it will be easier to find. When you come across a worked-out example, read and follow the example carefully. Do not just skim it. Try working out the example yourself on another sheet of paper. Also, work the **Now Try Exercises** that appear after each example. The Now Try Exercises are designed so that you have the opportunity to immediately apply new ideas. Make notes of anything that you do not understand to ask your instructor.

Do the Homework   *Two very important commitments that you must make to be successful in this course are to attend class and do your homework regularly.* Your assignments must be worked conscientiously and completely. Mathematics cannot be learned by observation. You need to practice what you have heard in class. By doing homework you truly learn the material.

Don't forget to check the answers to your homework assignments. Answers to the odd-numbered exercises are in the back of this book. In addition, the answers to all the Cumulative Review Exercises, Mid-Chapter Tests, Chapter Review Exercises, Chapter Practice Tests, and Cumulative Review Tests are provided. For the Mid-Chapter Tests, Chapter Practice Tests, and Cumulative Review Tests, the section where the material was first introduced is provided in brackets after each answer. Answers to the Group Activity Exercises are not provided because we want you to arrive at the answers as a group.

If you have difficulty with some of the exercises, mark them and do not hesitate to ask questions about them in class. You should not feel comfortable until you understand all the concepts needed to work every assigned problem.

When you do your homework, make sure that you write it neatly and carefully. Pay particular attention to copying signs and exponents correctly. Do your homework in a step-by-step manner. This way you can refer back to it later and still understand what was written.

Attend and Participate in Class   You should attend every class. Generally, the more absences you have, the lower your grade will be. Every time you miss a class, you miss important information. If you must miss a class, contact your instructor ahead of time and get the reading assignment and homework.

While in class, pay attention to what your instructor is saying. If you do not understand something, ask your instructor to repeat or explain the material. If you do not ask questions, your instructor will not know that you have a problem understanding the material.

In class, take careful notes. Write numbers and letters clearly so that you can read them later. It is not necessary to write down every word your instructor says. Copy down the major points and the examples that do not appear in the text. You should not be taking notes so frantically that you lose track of what your instructor is saying.

Study   Study in the proper atmosphere. Study in an area where you are not constantly disturbed so that your attention can be devoted to what you are reading. The area where you study should be well ventilated and well lit. You should have sufficient desk space to spread out all your materials. Your chair should be comfortable. You should try to minimize distractions while you are studying. You should not study for hours on end. Short study breaks are a good idea.

When studying, you should not only understand how to work a problem, you should also know why you follow the specific steps you do to work the problem. If you do not have an understanding of why you follow the specific process, you will not be able to solve similar problems.

Time Management   It is recommended that students spend at least 2 hours studying and doing homework for every hour of class time. Some students require more time than others. Finding the necessary time to study is not always easy. The following are some suggestions that you may find helpful.

1. Plan ahead. Determine when you will have time to study and do your homework. Do not schedule other activities for these time periods. Try to space these periods evenly over the week.
2. Be organized so that you will not have to waste time looking for your books, pen, calculator, or notes.
3. Use a calculator to perform tedious calculations.
4. When you stop studying, clearly mark where you stopped in the text.
5. Try not to take on added responsibilities. You must set your priorities. If your education is a top priority, as it should be, you may have to cut the time spent on other activities.
6. If time is a problem, do not overburden yourself with too many courses. Consider taking fewer credits. If you do not have sufficient time to study, your understanding and your grades in all of your courses may suffer.

③ Prepare for and Take Examinations

Study for an Exam   If you do some studying each day, you should not need to cram the night before an exam. If you wait until the last minute, you will not have time to seek the help you may need. To review for an exam,

1. Read your class notes.
2. Review your homework assignments.

3. Study the formulas, definitions, and procedures you will need for the exam.

4. Read the Avoiding Common Errors boxes and Helpful Hint boxes carefully.

5. Read the summary at the end of each chapter.

6. Work the review exercises at the end of each chapter. If you have difficulties, restudy those sections. If you still have trouble, seek help.

7. Work the Mid-Chapter Tests and the Chapter Practice Tests.

8. Rework quizzes previously given if the material covered in the quizzes will be included on the test.

9. Work the Cumulative Review Test if material from earlier chapters will be included on the test.

Take an Exam   Make sure that you get a good night's sleep the day before the test. If you studied properly, you should not have to stay up late the night before to prepare for the test. Arrive at the exam site early so that you have a few minutes to relax before the exam. If you need to rush to get to the exam, you will start out nervous and anxious. After you receive the exam, do the following:

1. Carefully write down any formulas or ideas that you want to remember.

2. Look over the entire exam quickly to get an idea of its length and to make sure that no pages are missing. You will need to pace yourself to make sure that you complete the entire exam. Be prepared to spend more time on problems worth more points.

3. Read the test directions carefully.

4. Read each problem carefully. Answer each question completely and make sure that you have answered the specific question asked.

5. Starting with number 1, work each question in order. If you come across a question that you are not sure of, do not spend too much time on it. Continue working the questions that you understand. After completing all other questions, go back and finish those questions you were not sure of. Do not spend too much time on any one question.

6. Attempt each problem. You may be able to earn at least partial credit.

7. Work carefully and write clearly so that your instructor can read your work. Also, it is easy to make mistakes when your writing is unclear.

8. Check your work and your answers if you have time.

9. Do not be concerned if others finish the test before you. Do not be disturbed if you are the last to finish. Use all your extra time to check your work.

4 Find Help

Use the Supplements   This text comes with many supplements. Find out from your instructor early in the semester which supplements are available and which might be beneficial for you to use. Reading supplements should never replace reading the text-book. Instead, supplements should enhance your understanding of the material. If you miss a class, you may want to review the video on the topic you missed before attending the next class.

The supplements that may be available to you are the Student's Solutions Manual, which works out the odd-numbered section exercises and all end-of-chapter exercises; the Section Lecture Videos, available in MyLab Math, which contain about 20 minutes of lecture per section and include additional examples; the Chapter Test Prep Videos, which present step-by-step solutions to every exercise in the each chapter's Practice Test and are available via MyLab Math.

Seek Help   One thing we stress with our own students is to *get help as soon as you need it!* Do not wait! In mathematics, one day's material is usually based on the previous day's material. So if you don't understand the material today, you may not be able to understand the material tomorrow.

Where should you seek help? There are often a number of places to obtain help on campus. You should try to make a friend in the class with whom you can study. Often you can help one another. You may wish to form a study group with other students in your class. Discussing the concepts and homework with your peers will reinforce your own understanding of the material.

You should not hesitate to visit your instructor when you are having problems with the material. Be sure you read the assigned material and attempt the homework before meeting with your instructor. Come prepared with specific questions to ask.

Often other sources of help are available. Many colleges have a mathematics laboratory or a mathematics learning center where tutors are available to help students. Ask your instructor early in the semester if any tutors are available, and find out where the tutors are located. Then use these tutors as needed.

5 Learn to Use a Calculator

Many instructors require their students to purchase and to use a calculator in class. You should find out as soon as possible which calculator, if any, your instructor expects you to use. If you plan on taking additional mathematics courses, you should determine which calculator will be required in those courses and consider purchasing that calculator for use in this course if its use is permitted by your instructor. Many instructors require a scientific calculator and many others require a graphing calculator.

In this book we provide information about both types of calculators. Always read and save the user's manual for whatever calculator you purchase. In the Using Your Graphing Calculator boxes, we will provide keystroke sequences for the TI-83 Plus and the TI-84 Plus graphing calculators. If you are using a different graphing calculator, you may need to read the calculator manual or go online to determine the correct keystroke sequences.

1.1 EXERCISE SET   MyLab Math

Do you know all of the following information? If not, ask your instructor as soon as possible.

1. What is your instructor's name?
2. What are your instructor's office hours?
3. Where is your instructor's office located?
4. How can you best reach your instructor?
5. Where can you obtain help if your instructor is not available?
6. What supplements are available to assist you in learning?
7. Does your instructor recommend or require a specific calculator? If so, which one?
8. When can you use a calculator? Can it be used in class, on homework, on tests?
9. What is your instructor's attendance policy?
10. Why is it important that you attend every class possible?
11. Do you know the name and phone number of a friend in class?
12. For each hour of class time, how many hours outside class are recommended for homework and studying?
13. List what you should do to be properly prepared for each class.
14. Explain how a mathematics textbook should be read.
15. Write a summary of the steps you should follow when taking an exam.
16. Having a positive attitude is very important for success in this course. Are you beginning this course with a positive attitude? It is important that you do!
17. You need to make a commitment to spend the time necessary to learn the material, to do the homework, and to attend class regularly. Explain why you believe this commitment is necessary to be successful in this course.

18. What are your reasons for taking this course?

19. What are your goals for this course?

20. Have you given any thought to studying with a friend or a group of friends? Can you see any advantages in doing so? Can you see any disadvantages in doing so?

1.2 Sets and Other Basic Concepts

1 Identify sets.

2 Identify and use inequalities.

3 Use set builder notation.

4 Determine the union and intersection of sets.

5 Identify important sets of numbers.

> **Variable**
>
> When a letter is used to represent various numbers it is called a **variable**.

For instance, if $t =$ the time, in hours, that a car is traveling, then t is a variable since the time is constantly changing as the car is traveling. We often use the letters x, y, z, and t to represent variables. However, other letters may be used.

If a letter represents one particular value it is called a **constant**. For example, if $s =$ the number of seconds in a minute, then s represents a constant because there are always 60 seconds in a minute. The number of seconds in a minute does not vary. In this book, letters representing both variables and constants are italicized.

The term **algebraic expression**, or simply **expression**, will be used often in the text. An expression is any combination of numbers, variables, exponents, mathematical symbols (other than equals signs), and mathematical operations.

1 Identify Sets

A **set** is a collection of objects. The objects in a set are called **elements** of the set. Sets are indicated by means of braces, { }, and are often named with capital letters. When the elements of a set are listed within the braces, as illustrated below, the set is said to be in **roster form**.

Set	Number of Elements
$A = \{a, b, c\}$	3
$B = \{\text{yellow, green, blue, red}\}$	4
$C = \{1, 2, 3, 4, 5\}$	5

The symbol $\in$ is used to indicate that an item is an element of a set. Since 2 is an element of set C we may write

$$2 \quad \in \quad C$$
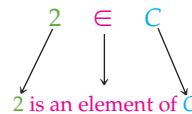
2 is an element of C

This is read "2 is an element of the set C."

A set may be finite or infinite. Sets A, B, and C each have a finite number of elements and are therefore *finite sets*. In some sets it is impossible to list all the elements. These are *infinite sets*. The following set, called the set of **natural numbers** or **counting numbers**, is an example of an infinite set.

$$N = \{1, 2, 3, 4, 5, \dots\}$$

> **Understanding Algebra**
>
> The *positive integers* are
>
> $1, 2, 3, 4, 5, 6, \dots$
>
> The *negative integers* are
>
> $-1, -2, -3, -4, -5, -6, \dots$

The three dots after the last comma are called an **ellipsis**. They indicate that the set continues on and on in the same manner.

Another important infinite set is the integers. The set of **integers** follows.

$$I = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$

Notice that the set of integers includes both positive and negative integers and the number 0.

If we write

$$D = \{1, 2, 3, 4, 5, \ldots, 163\}$$

we mean that the set continues in the same manner until the number 163. Set D is the set of the first 163 natural numbers. D is therefore a finite set.

A special set that contains no elements is called the **null set**, or **empty set**, written $\{ \ \}$ or $\varnothing$. For example, the set of students in your class under 3 years of age is the null or empty set.

2 Identify and Use Inequalities

Inequality Symbols

> $>$ is read "is greater than."
>
> $\geq$ is read "is greater than or equal to."
>
> $<$ is read "is less than."
>
> $\leq$ is read "is less than or equal to."
>
> $\neq$ is read "is not equal to."

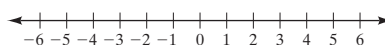Inequalities can be explained using the real number line (**Fig. 1.1**).

FIGURE 1.1

The number a is greater than the number b, $a > b$, when a is to the right of b on the number line (**Fig. 1.2**). We can also state that the number b is less than a, $b < a$, when b is to the left of a on the number line. The inequality $a \neq b$ means either $a < b$ or $a > b$.
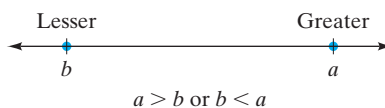

$a > b$ or $b < a$

FIGURE 1.2

EXAMPLE 1 Insert either $>$ or $<$ in the shaded area between the numbers to make each statement true.

a) 2 ▢ 6 **b)** 1 ▢ −7 **c)** −5 ▢ −4

Solution Indicate the location of the numbers in parts **a)**, **b)**, and **c)** on a number line as shown in **Figure 1.3**.
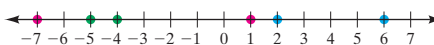
FIGURE 1.3

a) Because 2 is to the left of 6 on the number line, 2 is less than 6, and we write $2 < 6$.

b) Because 1 is to the right of −7 on the number line, 1 is greater than −7, and we write $1 > -7$.

c) Because −5 is to the left of −4 on the number line, −5 is less than −4, and we write $-5 < -4$.

Now Try Exercise 15

HELPFUL HINT

Remember that the symbol used in an inequality, if it is true, always points to the smaller of the two numbers.

Notation	Means
$x > 2$	x is any real number greater than 2.
$x \leq -3$	x is any real number less than or equal to -3.
$-4 \leq x < 3$	x is any real number greater than or equal to -4 and less than 3.

In the inequalities $x > 2$ and $x \leq -3$, the 2 and the -3 are called **endpoints**. In the inequality $-4 \leq x < 3$, the -4 and 3 are the endpoints. The solutions to inequalities that use either $<$ or $>$ do not include the endpoints, but the solutions to inequalities that use either $\leq$ or $\geq$ do include the endpoints. This is shown as follows:

Endpoint not included   Endpoint included
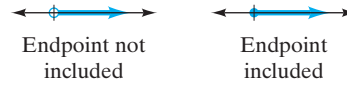
Below are three illustrations.

Inequality	Inequality Indicated on the Number Line
$x > 2$	
$x \leq -1$	
$-4 \leq x < 3$	

The word *between* indicates that the endpoints are not included in the answer. For example, the set of natural numbers between 2 and 6 is $\{3, 4, 5\}$. If we wish to include the endpoints, we can use the word *inclusive*. For example, the set of natural numbers between 2 and 6 inclusive is $\{2, 3, 4, 5, 6\}$.
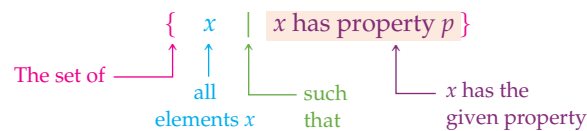
③ Use Set Builder Notation

A second method of describing a set is called **set builder notation**. An example of set builder notation is

$$E = \{x \,|\, x \text{ is a natural number greater than } 7\}$$

This is read "Set E is the set of all elements x, such that x is a natural number greater than 7." In roster form, this set is written

$$E = \{8, 9, 10, 11, 12, \dots\}$$

The general form of set builder notation is

$$\{ \ x \ | \ x \text{ has property } p \ \}$$

The set of — all elements x — such that — x has the given property

We often will use the variable x when using set builder notation, although any variable can be used.

Two condensed ways of writing set $E = \{x \,|\, x \text{ is a natural number greater than } 7\}$ in set builder notation follow.

$$E = \{x \,|\, x > 7 \text{ and } x \in N\} \quad \text{or} \quad E = \{x \,|\, x \geq 8 \text{ and } x \in N\}$$

The set $A = \{x \,|\, -3 < x \leq 4 \text{ and } x \in I\}$ is the set of integers greater than -3 and less than or equal to 4. The set written in roster form is $\{-2, -1, 0, 1, 2, 3, 4\}$. Notice that the endpoint -3 is not included in the set but the endpoint 4 is included.

How do the sets $B = \{x \,|\, x > 2 \text{ and } x \in N\}$ and $C = \{x \,|\, x > 2\}$ differ? Set B contains only the natural numbers greater than 2, that is, $\{3, 4, 5, 6, \dots\}$. Set C contains not only the natural numbers greater than 2 but also fractions and decimal numbers greater than 2. Since there is no smallest number greater than 2, this set cannot be written in roster form. We illustrate these two sets on the number line on the top of the next page. We have also illustrated two other sets.
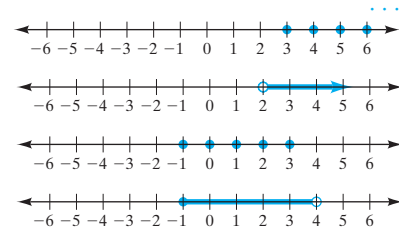
Set	**Set Indicated on the Number Line**
$\{x \mid x > 2 \text{ and } x \in N\}$	
$\{x \mid x > 2\}$	
$\{x \mid -1 \le x < 4 \text{ and } x \in I\}$	
$\{x \mid -1 \le x < 4\}$	

Another method of indicating inequalities, called *interval notation*, will be discussed in Section 2.5.

4   Determine the Union and Intersection of Sets

Just as *operations* such as addition and multiplication are performed on numbers, operations can be performed on sets. Two set operations are *union* and *intersection*.

Union of Two Sets

The **union** of set A and set B, written $A \cup B$, is the set of elements that belong to either set *A or* set *B*.

Because the word *or*, as used in this context, means belonging to set A or set B or both sets, the union is formed by combining, or joining together, the elements in set A with those in set B. If an item is an element in either set A, or set B, or in both sets, then it is an element in the union of the sets, $A \cup B$. If an element appears in both sets, we list it only once when we write the union of two sets.

Examples of Union of Sets

$$A = \{1,2,3,4,5\}, \qquad B = \{3,4,5,6,7\}, \qquad A \cup B = \{1,2,3,4,5,6,7\}$$
$$A = \{a,b,c,d,e\}, \qquad B = \{x,y,z\}, \qquad A \cup B = \{a,b,c,d,e,x,y,z\}$$

In set builder notation we can express $A \cup B$ as

Union

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Intersection of Two Sets

The **intersection** of set A and set B, written $A \cap B$, is the set of all elements that are common to both set *A and* set *B*.

Because the word *and*, as used in this context, means belonging to *both* set A and set B, the intersection is formed by using only those elements that are in both set A and set B. If an item is an element in only one of the two sets, then it is not an element in the intersection of the sets.

Examples of Intersection of Sets

$$A = \{1,2,3,4,5\}, \qquad B = \{3,4,5,6,7\}, \qquad A \cap B = \{3,4,5\}$$
$$A = \{a,b,c,d,e\}, \qquad B = \{x,y,z\}, \qquad A \cap B = \{ \ \} \text{ or } \varnothing$$